

BER OF OFDM

↓  
Bit Error Rate

↓  
Orthogonal Freq.  
Division Multiplexing

$$y = h \otimes x + w$$

$\nwarrow$   
 Gaussian Noise

FFT  
↓

$$Y(k) = H(k) X(k) + W(k)$$

$\nearrow$   
 $k^{\text{th}}$  FFT pt of  $[y(0), y(1), \dots, y(N-1)]$

$k^{\text{th}}$  FFT pt of  $[w(0), w(1), \dots, w(N-1)]$

$$W(k) = \sum_{l=0}^{N-1} w(l) e^{-j2\pi kl/N}$$

$k^{\text{th}}$  FFT pt of noise samples.

$w(l)$  — Gaussian IID  
zero mean

zero mean

$$E\{w(l)\} = 0 \quad E\{|w(l)|^2\} = \sigma^2$$

$$E\{w(l)w^*(\tilde{l})\} = 0$$

$$W(k) = \sum_{l=0}^{N-1} w(l) e^{-j2\pi kl/N}$$

$$E\{W(k)\} = E\left\{ \sum_{l=0}^{N-1} w(l) e^{-j2\pi kl/N} \right\}$$

$$= \sum_{l=0}^{N-1} \underbrace{E\{w(l)\}}_0 e^{-j2\pi kl/N}$$

$$= 0$$

$$\rightarrow E\{W(k)\} = 0.$$

$$E \{ |W(k)|^2 \} = ?$$

$$E \{ W(k) \cdot W^*(k) \}$$

$$= E \left\{ \left( \sum_{l=0}^{N-1} W(l) e^{j2\pi k l / N} \right) \left( \sum_{\tilde{l}=0}^{N-1} W(\tilde{l}) e^{-j2\pi k \tilde{l} / N} \right)^* \right\}$$

$$= E \left\{ \sum_{l=0}^{N-1} \sum_{\tilde{l}=0}^{N-1} \underbrace{W(l) W^*(\tilde{l})}_{e^{-j2\pi k (l - \tilde{l}) / N}} \right\}$$

$$= \sum_{l=0}^{N-1} \sum_{\tilde{l}=0}^{N-1} \underbrace{E\{W(l)W^*(\tilde{l})\}}_{\substack{=0 \text{ if } l \neq \tilde{l} \\ =\sigma^2 \text{ if } l = \tilde{l}}} e^{-j2\pi k(l-\tilde{l})/N}$$

$$= \sum_{l=0}^{N-1} \sigma^2 \cdot 1$$

$$= N\sigma^2$$

$$E\{|W(k)|^2\} = N\sigma^2$$

$W(k)$  ——— Gaussian

$$E \{ W(k) \} = 0$$

$$E \{ |W(k)|^2 \} = \underline{N\sigma^2}$$

↑  
Noise variance at  
output of FFT.

$H(k) = k^{\text{th}}$  FFT pt of

$$h(0) h(1) \dots h(L-1) \underbrace{00 \dots 0}_{N-L \text{ zeros.}}$$

$$H(k) = \sum_{l=0}^{L-1} h(l) e^{-j2\pi k l / N}$$

$$\downarrow E\{h(l)\} = 0$$

$$\downarrow E\{h(l) h^*(\tilde{l})\} = 0$$

"Uncorrelated Scattering"

$$E\{|h(l)|^2\} = 1$$

$h(l)$  are Rayleigh Fading

Complex Gaussian

$$E\{h(l)\} = 0$$

$$E\{|h(l)|^2\} = 1$$

Uncorrelated Scattering

$$E\{h(l)h^*(\tilde{l})\} = 0$$

if  $l \neq \tilde{l}$

$$H(k) = \sum_{l=0}^{L-1} h(l) e^{-j2\pi k l / N}$$

Complex Gaussian

Complex Gaussian

Rayleigh Fading

Zero mean

$$E\{H(k)\} = 0$$

$$E\{|H(k)|^2\} = \sum_{l=0}^{L-1} E\{|h(l)|^2\} = \sum_{l=0}^{L-1} 1 = L$$

$H(k)$

Rayleigh  
Fading

Zero mean

$$E\{H(k)\} = 0$$

$$E\{|H(k)|^2\} = L$$

$$Y(k) = H(k)X(k) + N(k)$$

Rayleigh  
Fading average  
power  $L$

Gaussian  
 $E\{|W(k)|^2\} = N_0$

$$E\{|X(k)|^2\} = P.$$

$$SNR_r = |H(k)|^2 \frac{P}{N_0}$$

$$SNR_r = \frac{|H(k)|^2 P}{N\sigma^2}$$

SNR at Receiver

$$\text{Average } \{SNR_r\} = \frac{LP}{N\sigma^2}$$

$$BER = \frac{1}{2} \left( 1 - \sqrt{\frac{L/N \text{ SNR}}{2 + L/N \text{ SNR}}} \right)$$

OFDM.

$$SNR = \frac{P}{\sigma^2} \rightarrow \text{Transmit SNR}$$

Example: BER of OFDM  
with  $L = 16$  channel Taps.

$N = 256$  sub carriers.

$$\text{SNR} = 35 \text{ dB} = \frac{P}{\sigma^2}.$$

$$\text{BER} = ?$$

$$10 \log_{10} \text{SNR} = 35$$

$$\text{SNR} = 10^{3.5}$$

$$L = 16.$$

$$N = 256$$

$$\text{BER} = \frac{1}{2} \left( 1 - \sqrt{\frac{L/N \text{SNR}}{2 + L/N \text{SNR}}} \right)$$

$$= \frac{1}{2} \left( 1 - \sqrt{\frac{\frac{16}{256} \times 10^{3.5}}{2 + \frac{16}{256} \times 10^{3.5}}} \right)$$

$$\text{BER} = 2.5 \times 10^{-3}$$


---